ON THE STABILITY OF THE FIRST ORDER LINEAR RECURRENCE IN TOPOLOGICAL VECTOR SPACES

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ABSTRACT. Suppose that $\mathcal{X}$ is a sequentially complete Hausdorff locally convex space over a scalar field $\mathbb{K}$, V is a bounded subset of $\mathcal{X}$, $(a_n)_{n\geq 0}$ is a sequence in $\mathbb{K}\setminus\{0\}$ with the property $\liminf\limits_{n\to\infty} |a_n| > 1$ and $(b_n)_{n\geq 0}$ is a sequence in $\mathcal{X}$. We show that for every sequence $(x_n)_{n\geq 0}$ in $\mathcal{X}$ satisfying

$$x_{n+1} - a_n x_n - b_n \in V \quad (n \geq 0)$$

there exists a unique sequence $(y_n)_{n\geq 0}$ satisfying the recurrence $y_{n+1} = a_n y_n + b_n$ $(n \geq 0)$ and for every q with $1 < q < \liminf\limits_{n\to\infty} |a_n|$, there exists $n_0 \in \mathbb{N}$ such that

$$x_n - y_n \in \frac{1}{q-1}\overline{conv(V^b)} \quad (n \geq n_0).$$

1. Introduction

The stability problem of functional equations was originally raised by Ulam [19] in 1940 on a talk at Wisconsin University. The problem posed by Ulam was the following: "Under what conditions does there exist an additive mapping near an approximately additive mapping?" The first answer to the question was given by Hyers in the case of Banach spaces [7]. After Hyers' result many papers dedicated to this topic extending Ulam's problem to other

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