Solving Nonlinear Integral Equations in the Urysohn form by Newton-Kantorovich Quadrate Method

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Abstract

The Newton-Kantorovich method is a well known method for solving nonlinear integral equations. This method attempts to solve a sequence of linear integral equations. In this article, we develop a new method which is a combination of the Newton-Kantorovich and quadrature methods. The new method solves the nonlinear integral equations of the Urysohn form in a systematic procedure. Some numerical examples are provided and the obtained numerical approximations are compared with the corresponding exact solutions.

Keywords: Nonlinear integral equation; Newton-Kantorovich method; Quadrature; Urysohn integral equation

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1. Introduction and preliminaries

One kind of the nonlinear integral equation is the nonlinear integral equation in the Urysohn form. This kind of integral equations is defined in the following general form:

$$y(x) = f(x) + \int_{\Omega} K(x,t,y(t))dt\,,\ a \le x \le b. \tag{1.1}$$

Depending on $\Omega = (a,x)$ or $\Omega = (a,b)$ equation (1.1) is named to a nonlinear Volterra integral equation or a nonlinear Fredholm integral equation, respectively.

To approximate the right-hand integral in (1.1), we use the usual quadrature methods similar to the ones used to approximate the linear integral equations that lead to the following nonlinear systems for Fredholm and Volterra equations, respectively. For further information on quadrature methods in this respect, see [3- 13].

$$y(x_i) = f(x_i) + \sum_{j=0}^{n} w_j K(x_i, x_j, y(x_j))\,,\ i = 0,1,\dots,n \tag{1.2}$$

$$\begin{cases} y(x_0) = f(x_0) \\ y(x_i) = f(x_i) + \sum_{j=0}^{i} w_{ij} K(x_i, x_j, y(x_j))\,,\ i = 1,2,\dots,n, \end{cases} \tag{1.3}$$

where w_{ij} s and w_j s are weights of the integration formula.

We recall that in the Newton-Kantorovich method, we consider an initial solution to $y(x)$, say $y_0(x)$. Then by using the following iteration method, we solve the following sequence of linear integral equations instead of a nonlinear integral equation. For further information on the Newton-Kantorovich method, see [1, 2 and 14].

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