A GRÜSS INEQUALITY FOR n-POSITIVE LINEAR MAPS

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Abstract. Let $\mathscr{A}$ be a unital C^*-algebra and let $\Phi : \mathscr{A} \to \mathbb{B}(\mathscr{H})$ be a unital n-positive linear map between C^*-algebras for some $n \geq 3$. We show that

$$\|\Phi(AB) - \Phi(A)\Phi(B)\| \leq \Delta(A, \|\cdot\|)\,\Delta(B, \|\cdot\|)$$

for all operators $A, B \in \mathscr{A}$, where $\Delta(C, \|\cdot\|)$ denotes the operator norm distance of C from the scalar operators.

1. Introduction

Let $\mathbb{B}(\mathscr{H})$ stand for the algebra of all bounded linear operators on a complex Hilbert space $(\mathscr{H}, \langle\cdot,\cdot\rangle)$, let $\|\cdot\|$ denote the operator norm and let I be the identity operator. For self-adjoint operators A, B the order relation $A \leq B$ means that $\langle A\xi, \xi\rangle \leq \langle B\xi, \xi\rangle$ $(\xi \in \mathscr{H})$. In particular, if $0 \leq A$, then A is called positive. If $\dim \mathscr{H} = k$, we identify $\mathbb{B}(\mathscr{H})$ with the algebra $\mathscr{M}_k$ of all $k \times k$ matrices with entries in $\mathbb{C}$.

Let $\Delta(C, \|\cdot\|) = \inf_{\lambda \in \mathbb{C}} \|C - \lambda I\|$ be the $\|\cdot\|$-distance of C from the scalar operators. It is known that $\Delta(C, \|\cdot\|) \leq \|C\|$ and $\Delta(C, \|\cdot\|) = c(C)$ for any normal operator C, where $c(C)$ denotes the radius of the smallest disk in the complex plane containing the spectrum $\sigma(C)$ of C; see [14].

A linear map $\Phi : \mathscr{A} \to \mathscr{B}$ between C^*-algebras is said to be *positive* if $\Phi(A) \geq 0$ whenever $A \geq 0$. Every positive linear map Φ satisfies $\Phi(A^*) = \Phi(A)^*$ for all A. We say that Φ is unital if $\mathscr{A}, \mathscr{B}$ are unital C^*-algebras and

2010 *Mathematics Subject Classification.* Primary 46L07; secondary 47A12, 47L25.

Key words and phrases. Grüss inequality, operator inequality, positive operator, C^*-algebra, completely positive map, n-positive map.