



A CHARACTERIZATION OF INNER PRODUCT SPACES RELATED TO THE p -ANGULAR DISTANCE

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ABSTRACT. In this paper we present a new characterization of inner product spaces related to the p -angular distance. We also generalize some results due to Dunkl, Williams, Kirk, Smiley and Al-Rashed by using the notion of p -angular distance.

1. INTRODUCTION

In 1935, Fréchet [9] gave a geometric characterization of inner product spaces. In the same year, Jordan and von Neumann [12] characterized inner product spaces as normed linear spaces satisfying the parallelogram law. In 1943, Ficken showed that a normed linear space is an inner product space if and only if a reflection about a line in any two-dimensional subspace is an isometric mapping. In 1947, Lorch presented several characterizations of inner product spaces. Since then the problem of finding necessary and sufficient conditions for a normed space to be an inner product space has been investigated by many mathematicians by considering some types of orthogonality or some geometric aspects of underlying spaces. Some

2010 *Mathematics Subject Classification*. Primary 46C15; Secondary 46B20, 46C05.

Key words and phrases. inner product space; characterization of inner product spaces; p -angular distance; Dunkl-Williams inequality.