Information Measures via Copula Functions

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Abstract

In applications of differential geometry to problems of parametric inference, the notion of divergence is often used to measure the separation between two parametric densities. Among them, in this paper, we will verify measures such as Kullback-Leibler information, J-divergence, Hellinger distances, α-Divergence,... and so on. Properties and results related to distance between probability distributions derived via copula functions. Some inequalities are obtained in view of the dependence and information measures.

Keywords: Information Measures, Fisher Information, Kullback-Leibler Information, Hellinger Distance, α-Divergence.
MSC 2000: Primary 94A15; Secondary 60E05.

1 Introduction

The study of copulas and the role they play is important in probability, statistics and stochastic processes. Sklar (1959) provided a uniform representation of bivariate distribution F on the unit square and defined copula based on it. Many research papers and monographs due to copula aspect are published after Sklar (1959), such as Nelsen (2006), Joe (1997), Cherubini et al. (2004) and Mari and Kotz (2004) and their references in. Frees and Valdez (1998) introduced the concept of copulas as a tool for understanding relationships among multivariate outcomes. Also, dependence and copulas have linked with each other.

The concept of the entropy originated in the nineteenth century by C. E. Shannon (1948). During the last sixty years or so, a number of publications discussing and extending Shannon's original work have appeared. Among them, Ali Ahmed et al. (1989), Darbellay and Vajda (1998, 2000), Dragomir (2003), Blyth (1994), Torkkola (2003), Kapur (1989,1994), Borovkov (1998), Kagan, Linnik and Rao (1973) and Kullback (1959) are mentioned in this research. Ma and Sun (2008) and Calsaverini and Vicente (2009) are two recent notes related to the links between copula and information measures.

In this paper, various measures are obtained in view of copulas for bivariate distributions. Properties of information measures and their links with copula is another direction of this research.

2 Preliminaries and some information measures

Let $(\Omega, \mathcal{B}, \mu)$ be a measure space and f be a measurable function from Ω to $[0, \infty)$, such that $\int_\Omega f d\mu = 1$. The Shannon entropy (or simply the entropy) of f relative to μ, is defined by

$$H(f, \mu) = -\int_\Omega f \ln f d\mu, \text{ (with } f \ln f = 0 \text{ if } f = 0), \tag{1}$$

and assumed to be defined for which $f \ln f$ is integrable. If X is an r.v. with pdf f, then we refer to H as the entropy of X and denotes it by the notation H_X as well. In the case μ is a version of counting

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