ON THE FREE RESOLUTIONS OF LOCAL COHOMOLOGY MODULES WITH RESPECT TO AN IDEAL GENERATED BY A U.S.D-SEQUENCE

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ABSTRACT. Let $\mathfrak{a}$ be an almost complete intersection ideal of a commutative Noetherian local ring R and r be the number of elements of a minimal generating set of $\mathfrak{a}$. Suppose that the ith local cohomology module $H^i_{\mathfrak{a}}(R)$ is finitely generated for all $i < r$. We show that there exists a sequence $\mathrm{x} = x_1, \ldots, x_r$ of elements in $\mathfrak{a}$ which is both an $\mathfrak{a}$-filter regular and u.s.d-sequence on R and

$$\Omega^{r-1}_R(H^{r-1}_{\mathfrak{a}}(R)) \cong \Omega^{r+1}_R(R/(\mathrm{x}))$$

where, for an R-module M, $\Omega^i_R(M)$ is the ith syzygy of M.

1. INTRODUCTION

Let R be a commutative ring and M be an R-module. For an ideal $\mathfrak{a}$ of R, we denote the ith local cohomology functor with respect to $\mathfrak{a}$ by $H^i_{\mathfrak{a}}(-)$. Also, for a minimal free resolution F of M, we set the ith syzygy of M by $\Omega^i_R(M)$, that is $\mathrm{Coker}\partial^F_{i+1}$.

There have been some works on the study of the syzygies of different modules. But there is not many paper concerning the syzygies of local cohomology modules. In this paper, under certain circumstances, we obtain some syzygies of local cohomology modules of ideal generated by u.s.d-sequence $x_1, \ldots, x_n$, in terms of the syzygies of $R/(x_1, \ldots, x_n)$. Clearly, if R is a d-dimensional Cohen-Macaulay local ring and $\mathrm{x} = x_1, \ldots, x_d$ is a system of parameters for R, a minimal free resolution $R/(\mathrm{x})$ is determined by a Koszul complex of d elements. Recently, in [8], Rahmati proved that if R is a d-dimensional local ring with maximal ideal $\mathfrak{m}$, $d - \mathrm{depth}R \leq 1$ and $H^{d-1}_{\mathfrak{m}}(R)$ is finitely generated, then there exists an integer n such that for every system of parameters x for R contained in $\mathfrak{m}^n$,

$$\Omega^{d-1}_R(H^{d-1}_{\mathfrak{m}}(R)) \cong \Omega^{d+1}_R(R/(\mathrm{x})).$$

In this paper, by using natural generalizations of regular sequence which are called d-sequence and filter regular sequence, we show the following theorem.

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