



LOCAL COHOMOLOGY MODULES, SERRE SUBCATEGORIES AND DERIVED FUNCTORS OF TORSION FUNCTORS

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ABSTRACT. In this research, by using filter regular sequences, we obtain some exact sequences of right or left derived functors of local cohomology modules. Then we use them to gain some conditions under which a right or left derived functor of some special functors over local cohomology modules belongs to a Serre subcategory. These results can conclude some generalizations of previous results in this context or regain some of them.

1. INTRODUCTION

Throughout this paper, R will denote a commutative Noetherian ring with non-zero identity, \mathfrak{a} an ideal of R and M a weakly Laskerian R -module. Also, F and T are two covariant functors so that there exists an ideal \mathfrak{b} of R containing \mathfrak{a} such that for each R -module L , $F(L)$ and $T(L)$ are \mathfrak{b} -torsion.

Local cohomology theory which is initially defined and studied in 1968 by Grothendieck, [15], is a useful tool for attacking problems in commutative algebra and algebraic geometry. After publishing an article of Huneke in 1982 [19], studying some problems such as Noetherianness, Artinianness, vanishing and finiteness of the set of associated prime ideals of these modules, have been taken into the consideration of a lot of researchers (cf. [8], [28], [17], [29], [20], [23], [13], [21], [22], [1] and [2]). Also, some generalizations of these properties were proposed and studied by some authors (cf. [18], [5], [34], [24] and [3]).

In the second section of this paper, we obtain some exact sequences of right derived functors of functor F (or left derived functors of functor T) over an arbitrary local cohomology module $H_{\mathfrak{a}}^m(M)$. To do this we use a natural generalization of the concept of regular sequences which is called filter regular sequences. Let \mathcal{S} be a Serre subcategory of the category of R -modules containing the zero module. In section three, we obtain some conditions under which a right (or left) derived functor of some special functors over local cohomology modules belongs to \mathcal{S} . Also

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