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Function Valued Metric Spaces

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Abstract. In this paper we introduce the notion of an $\mathcal{F}$-metric, as a function valued distance mapping, on a set X and we investigate the theory of $\mathcal{F}$-metric spaces. We show that every metric space may be viewed as an $\mathcal{F}$-metric space and every $\mathcal{F}$-metric space (X, δ) can be regarded as a topological space (X, τ_δ). In addition, we prove that the category of the so-called extended $\mathcal{F}$-metric spaces properly contains the category of metric spaces. We also introduce the concept of an $\bar{\mathcal{F}}$-metric space as a completion of an $\mathcal{F}$-metric space and, as an application to topology, we prove that each normal topological space is $\bar{\mathcal{F}}$-metrizable.

1 Introduction

The celebrated paper of Zadeh [15], motivated some authors to define and discuss some notions of a fuzzy metric on a set and fuzzy norm on a linear space. A probabilistic metric space is a fuzzy generalization of metric spaces where the distance is no longer defined on positive real numbers, but on distribution functions. For an account on probabilistic metric spaces the reader is referred to the book [13]. Katsaras [6] in 1984 introduced a concept of a fuzzy norm. Later, Felbin [4] introduced an idea of a fuzzy norm on a linear space by assigning a fuzzy real to each element of the fuzzy linear space so that the corresponding metric associated to this fuzzy norm is of the Kaleva type [5] fuzzy metric. In 2003, following [3], Bag and Samanta in [1] and [2], introduced and studied an idea of a fuzzy norm on a linear space in such a manner that its corresponding fuzzy metric is of Kramosil and Michalek type [8]. All of these can be assumed as norms whose values are mappings.

On the other hand, if $\mathcal{A}$ is a C^*-algebra then a Hilbert $\mathcal{A}$-module is a right $\mathcal{A}$-module $\mathcal{E}$ (which is at the same time a complex vector space) equipped with an $\mathcal{A}$-valued inner product. The reader is referred to [9, 10, 14] and [12] for further information on Hilbert C^*-modules. If $\mathcal{A}$ is a commutative C^*-algebra then an $\mathcal{A}$-valued inner product is a function valued inner product.

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Keywords: Function valued metric; positive element; strictly positive; $\mathcal{F}$-completeness; $\mathcal{F}$-metric space; allowance set; $\mathcal{F}$-Cauchy; $\mathcal{F}$-completion; $\mathcal{F}$-metrizable.

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