Point Prediction of Future Order Statistics from Exponential Distribution

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Abstract

Two sample point prediction is considered for two parameter exponential distribution. Several point predictors such as best unbiased predictor, best invariant predictor and maximum likelihood predictor are obtained for future order statistics based on observed record values in two cases, location parameter is known and unknown. These predictors are compared in the sense of their mean squared prediction errors. Finally, some numerical results are given to illustrate the proposed procedures.

Keywords: BLIP; BLUP; MLP; MSPE; Record values.

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1 Introduction and preliminaries

Let $X_1, X_2, \ldots,$ be an infinite sequence of random variables and $Z_n = \max\{X_1, \ldots, X_n\}$ for $n \geq 1$, then an observation X_j is called an upper record value if $Z_j > Z_{j-1}, j > 1$. For a detailed review on applications of record values, see Arnold et al. (1998). One application of record values arises in association with a minimal repair process. Such a process can be explained briefly as follows: Consider the sequence of failure times of a technical system with one repairable component. After each failure, this component will immediately be repaired and replaced into its prior condition. The times of repairs and replacements are very low and therefore neglected. If F denotes the cumulative distribution function (cdf) of the lifetime of that component, then the minimal repair times possess the same joint distribution as record values based on F, (For more details see for example Shaked and Shanthikumar, 1994). Record values also have the same distribution as epoch times of some non-homogeneous Poisson process (NHPP) [See e.g. Pellerey et al. (2000) for more details].

Next, let $Y_1, \ldots, Y_m$ be a sample of size m, independent of the X_n-sequence, and $Y_{j:m}, 1 \leq j \leq m$ be the corresponding jth order statistic. Order statistics arise in many practical situations as well as reliability of systems. It is well-known that a system is called a k-out-of-m system if it consists of m components functioning satisfactorily provided that at least $k(\leq m)$ components function. If the lifetimes of the components are independently distributed, then the lifetime of the system coincides with that of the $(m-k+1)$th order statistic from the underlying distribution. Therefore, order statistics play a key role in studying the lifetimes of such systems. See Arnold et al. (1992) and David and Nagaraja (2003) for more details concerning the applications of order statistics.

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