



REVERSE CAUCHY-SCHWARZ TYPE INEQUALITIES IN PRE-INNER PRODUCT C^* -MODULES

J.I.FUJII¹, M.FUJII², M.S.MOSLEHIAN³, J.E.PEČARIĆ⁴ AND Y.SEO⁵

ABSTRACT. In the framework of pre-inner product C^* -modules, we show several reverse Cauchy-Schwarz type inequalities of additive and multiplicative types, by using some ideas in N. Elezović et al. [Math. Inequal. Appl. 8 (2005), no.2, 223–231]. We apply our results to give Klamkin-McLenaghan, Shisha-Mond and Cassels type inequalities. We also present a Grüss type inequality.

1. INTRODUCTION

A Hilbert C^* -module is a generalization of a Hilbert space in which the inner product takes its values in a C^* -algebra instead of the complex numbers. The theory of Hilbert C^* -modules is different from that of Hilbert spaces, for example, not any bounded linear operator between Hilbert C^* -modules is adjointable and not any closed submodule of a Hilbert C^* -module is complemented, see [11].

The theory of Hilbert C^* -modules over commutative C^* -algebras was first appeared in a work of Kaplansky [9] in 1953. The research on this subject started in 1970's independently by Paschke [17] and Rieffel [18] and since then it has grown rapidly and has played significant roles in the theory of operator algebras and noncommutative geometry.

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