OPERATOR ACZEL INEQUALITY

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ABSTRACT. We establish several operator versions of the classical Aczel inequality. One of operator versions deals with the weighted operator geometric mean and another is related to the positive sesquilinear forms. Some applications including the unital positive linear maps on C^*-algebras and the unitarily invariant norms on matrices are presented.

1. INTRODUCTION

Let $\mathbb{B}(\mathscr{H})$ denote the algebra of all bounded linear operators acting on a complex Hilbert space $(\mathscr{H}, \langle\cdot,\cdot\rangle)$ and I is the identity operator. In the case where $\dim \mathscr{H} = n$, we identify $\mathbb{B}(\mathscr{H})$ with the full matrix algebra $M_n(\mathbb{C})$ of all $n \times n$ matrices with entries in the complex field $\mathbb{C}$. An operator $A \in \mathbb{B}(\mathscr{H})$ is called positive (positive-semidefinite for matrices) if $\langle A\xi, \xi\rangle \geq 0$ holds for every $\xi \in \mathscr{H}$ and then we write $A \geq 0$. For $A, B \in \mathrm{B}(\mathcal{H})$, we say $A \leq B$ if $B - A \geq 0$. Let f be a continuous real valued function defined on an interval J. The function f is called operator decreasing if $B \leq A$ implies $f(A) \leq f(B)$ for all A, B with spectra in J. A function f is said to be operator concave on J if

$$\lambda f(A) + (1-\lambda)f(B) \leq f(\lambda A + (1-\lambda)B)$$

for all $A, B \in \mathbb{B}(\mathscr{H})$ with spectra in J and all $\lambda \in [0,1]$.

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