



Small loop spaces and covering theory of non-homotopically Hausdorff spaces

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Abstract

This paper is devoted to spaces that are not homotopically Hausdorff and study their covering spaces. We introduce the notion of small covering and prove that every small covering of X is the universal covering in the categorical sense. Also, we introduce the notion of semi-locally small loop space which is the necessary and sufficient condition for existence of universal covering for non-homotopically Hausdorff spaces, equivalently existence of small covering spaces. Moreover, we prove that for semi-locally small loop spaces, X is a small loop space if and only if every covering of X is trivial if and only if $\pi_1^{top}(X)$ is an indiscrete topological group.

Keywords: Covering space, Small loop group, Small loop space, Semi-locally small loop space, Homotopically Hausdorff space.

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1. Introduction and motivation

We recall that a continuous map $p : \tilde{X} \rightarrow X$ is a *covering* of X , and \tilde{X} is called a *covering space* of X , if for every $x \in X$ there exists an open subset U of X with $x \in U$ such that U is *evenly covered* by p , that is, $p^{-1}(U)$ is a disjoint union of open subsets of \tilde{X} each of which is mapped homeomorphically onto U by p .

In the classical covering theory, one assumes that X is, in addition, connected, locally path connected, semi-locally simply connected and wishes to classify all path connected covering spaces of X and to find among them the *universal covering* in

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