SCHATTEN p-NORM INEQUALITIES RELATED TO AN EXTENDED OPERATOR PARALLELOGRAM LAW

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ABSTRACT.

Let $\mathcal{C}_p$ be the Schatten p-class for $p > 0$. Generalizations of the parallelogram law for the Schatten 2-norms have been given in the following form: If $\mathbf{A} = \{A_1, A_2, \dots, A_n\}$ and $\mathbf{B} = \{B_1, B_2, \dots, B_n\}$ are two sets of operators of $\mathcal{C}_2$

$$\sum_{i,j=1}^{n} \|A_i - A_j\|_2^2 + \sum_{i,j=1}^{n} \|B_i - B_j\|_2^2 = 2\sum_{i,j=1}^{n} \|A_i - B_j\|_2^2 - 2\left\|\sum_{i=1}^{n}(A_i - B_i)\right\|_2^2.$$

In this paper, we give a generalization and a complementary inequality of the inequality above. Moreover, we present some related inequalities for three sets of operators.

1. INTRODUCTION

Let $\mathbb{B}(\mathcal{H})$ be the algebra of all bounded linear operators on a separable complex Hilbert space $\mathcal{H}$ endowed with inner product $\langle \cdot, \cdot \rangle$. We denote the absolute value of $A \in \mathbb{B}(\mathcal{H})$ by $|A| = (A^*A)^{1/2}$.

Let $A \in \mathbb{B}(\mathcal{H})$ be a compact operator and let $0 < p < \infty$. The Schatten p-norm (p-quasi-norm) for $1 \leq p < \infty$ ($0 < p < 1$) is defined by $\|A\|_p = (\mathrm{tr}|A|^p)^{1/p}$, where tr is the usual trace functional for $p > 0$. For $p > 0$, the

2010 *Mathematics Subject Classification.* Primary 47A63; Secondary 46C15, 47A30, 47B10, 47B15, 15A60.

Key words and phrases. Schatten p-norm; norm inequality; parallelogram law; inner product space; Clarkson inequality.