



ON THE MATLIS DUALS OF LOCAL COHOMOLOGY MODULES AND MODULES OF GENERALIZED FRACTIONS

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ABSTRACT. Let (R, \mathfrak{m}) be a commutative Noetherian local ring with non-zero identity, \mathfrak{a} a proper ideal of R and M a finitely generated R -module with $\mathfrak{a}M \neq M$. Let $D(-) := \text{Hom}_R(-, E)$ be the Matlis dual functor, where $E := E(R/\mathfrak{m})$ is the injective hull of the residue field R/\mathfrak{m} . In this paper, by using a complex which involves modules of generalized fractions, we show that, if x_1, \dots, x_n is a regular sequence on M contained in \mathfrak{a} , then $H_{(x_1, \dots, x_n)R}^n(D(H_{\mathfrak{a}}^n(M)))$ is a homomorphic image of $D(M)$, where $H_{\mathfrak{b}}^i(-)$ is the i -th local cohomology functor with respect to an ideal \mathfrak{b} of R . By applying this result, we study some conditions on a certain module of generalized fractions under which $D(H_{(x_1, \dots, x_n)R}^n(D(H_{\mathfrak{a}}^n(M)))) \cong D(D(M))$.

1. INTRODUCTION

For an ideal \mathfrak{a} of a commutative Noetherian local ring (R, \mathfrak{m}) , we denote the n -th local cohomology functor with respect to \mathfrak{a} by $H_{\mathfrak{a}}^n(-)$ and the Matlis dual functor $\text{Hom}_R(-, E)$ by $D(-)$, where E is the injective hull of the field R/\mathfrak{m} . Also, for a sequence $\underline{x} := x_1, \dots, x_n$ of elements of R , we use $\underline{x}R$ or $(x_1, \dots, x_n)R$ to denote the ideal $\sum_{i=1}^n x_i R$ of R .

Recently, there has been some work on modules $D(H_{\mathfrak{a}}^n(R))$ and $H_{\underline{x}R}^n(D(H_{\mathfrak{a}}^n(R)))$, where $\underline{x} := x_1, \dots, x_n$ is a sequence of elements of \mathfrak{a} , and on some problems related to these modules (see for example conjecture (*) in [3] and [4], and question 3.8 in [5]). Also, Hellus and Stückrad, in [5], showed that studying the Matlis dual of local cohomology modules is a useful tool for the description of the endomorphism ring of local cohomology modules. Moreover, they raised the following question: if R is a commutative Noetherian complete local ring and $\underline{x} := x_1, \dots, x_n$ is a regular sequence on R contained in \mathfrak{a} , when exactly is $J_{\underline{x}, \mathfrak{a}, R} := D(H_{\underline{x}R}^n(D(H_{\mathfrak{a}}^n(R))))$ zero?

On the other hand, Sharp and Zakeri, in [12], over an arbitrary commutative ring, introduced the concept of modules of generalized fractions. It was shown

2000 *Mathematics Subject Classification.* 13D45, 13D07, 13B30.

Key words and phrases. local cohomology module, Matlis dual functor, module of generalized fractions, filter regular sequence.

This research was supported by a grant from Ferdowsi University of Mashhad (No. MP87101KHA).

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