CHAPTER (I)

STABILITY OF A GENERALIZED JENSEN EQUATION ON RESTRICTED DOMAINS

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ABSTRACT. In this paper, we establish the conditional stability of the generalized Jensen functional equation $f(ax+by) = ag(x)+bh(y)$ on various restricted domains such as inside balls, outside balls, and punctured spaces. In addition, we prove the orthogonal stability of this equation and study orthogonally generalized Jensen mappings on balls in inner product spaces.

1. INTRODUCTION

It is known that the problem of stability of functional equations originated from the following question of Ulam [illegible] posed in 1940: "Given an approximately linear mapping f, when does a linear mapping T estimating f exist?" In the next year, Hyers [9] gave an affirmative answer to the question of Ulam in the context of Banach spaces. The theorem of Hyers was extended by T. Aoki [1] for additive mappings in 1950 and by Th.M. Rassias [25] for linear mappings in 1978 by considering the unbounded Cauchy difference $\|f(x+y)-f(x)-f(y)\| \le \varepsilon(\|x\|^p + \|y\|^p)$, where $\varepsilon > 0$ and $p \in [0,1)$ are constants. The paper [25] of Th.M. Rassias has provided a lot of influence in the development of what we now call Hyers–Ulam–Rassias stability of functional equations. In 1994, another generalization, the so-called generalized Hyers–Ulam–Rassias stability, was obtained by Găvruta [illegible]. During the last decades several stability problems of functional equations have been investigated. The reader is referred to [3, 5, 6, 7, 10, 11, 13, 16, 17, 18, 26]

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