NORM CONTINUITY OF QUASI-CONTINUOUS MAPPINGS INTO $C_p(X)$ AND PRODUCT SPACES

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Abstract. A compact space X is said to have the $\mathcal{NQ}$ property if for every α-favorable space A and every quasi-continuous function $\varphi : A \to C_p(X)$, there is a dense G_δ subset D of A such that φ is norm continuous at each point of D. We give a game theoretic proof to show that the property $\mathcal{NQ}$ is closed under arbitrary product.

1. Introduction

Let $C_p(X)$ denote the space of continuous functions on a compact set X equipped with the topology of pointwise convergence. We say that the compact space X has the $\mathcal{NQ}$ *property* if every quasi-continuous mapping φ from an α-favorable space A to $C_p(X)$ is norm continuous at each point of a dense G_δ subset of A.

In 1985, Talagrand [14], provided an example of a compact space X which does not have the property $\mathcal{NQ}$. In [8, Theorem 1] it was shown that if $C_p(X)$ is sigma-fragmented by the norm, then X has the property $\mathcal{NQ}$. It follows that the class of all compact spaces with the property $\mathcal{NQ}$ contains a wide variety of compact spaces such as Corson compacts, Helly compacts and scattered compact spaces X with $X^{(\omega_1)} = \emptyset$, see [5, 6, 10].

Bouziad [1, 2] showed that the class of co-Namioka spaces is stable under arbitrary products. Using the idea of Bouziad [2] and a result due to Jayne et al., [6, Theorem 4.4], Namioka and Pol [12] proved that the class of compact spaces X with sigma-fragmentable $C_p(X)$ is closed under arbitrary product. The ideas in [1] and [2] were used by Mykhaylyuk [11] to prove that the product of an arbitrary family of compact Kempisty spaces is again a Kempisty space.

Kenderov et al.[8, 9] obtained a game characterization for the property $\mathcal{NQ}$. In the present paper, we use this characterization to show that $X = \prod_{\lambda\in\Lambda} X_\lambda$ equipped with the

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