NON-ARCHIMEDEAN STABILITY OF QUADRATIC EQUATIONS

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ABSTRACT. We use fixed point method to study the Hyers-Ulam-Rassias stability of the quadratic equation in non-Archimedean normed spaces. A few applications of our result will be illustrated. We will also give an example to show that some results in stability and quadratic mappings in real normed spaces are not valid in non-Archimedean normed spaces.

1. INTRODUCTION

In mathematical analysis, we may meet the following stability problem:
Assume that a function satisfies a functional equation approximately according to some convention. Is it then possible to find near this function a function satisfying the equation accurately?
In 1940, S. M. Ulam [26] posed the first stability problem. In the next year, D.H. Hyers [8] gave a partial affirmative answer to the question of Ulam. The theorem of Hyers was generalized by T. Aoki [2] for additive mappings and by Th.M. Rassias [22] for linear mappings by considering an unbounded Cauchy difference. The paper of Th.M. Rassias has provided a lot of influence in the development of what we now call Hyers-Ulam-Rassias stability of functional equations. We refer the interested readers for more information on such problems to, e.g., In 1994, a generalization of Rassias' theorem was obtained by Găvruta [3] by replacing the bound $\varepsilon(\|x\|^p + \|y\|^p)$ by a general control function $\varphi(x, y)$. Several stability results have been recently obtained for various equations, also for mappings with more general domains and ranges (see [4, 9, 14, 10]). In [1], the authors showed that if $f : Q_p \to \mathbb{R}$ is a continuous mapping such that for some $\varepsilon > 0$,

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