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# STABILITY OF THE MONOMIAL FUNCTIONAL EQUATION IN QUASI NORMED SPACES

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ABSTRACT. Let $X$ be a linear space and $Y$ be a complete quasi $p$-norm space. We will show that for each function $f : X \to Y$, which satisfies the inequality

$$||\Delta_x^n f(y) - n!f(x)|| \leq \varphi(x, y),$$

for suitable control function $\varphi$, there is a unique monomial function $M$ of degree $n$ which is a good approximation for $f$ in such a way that the continuity of $t \mapsto f(tx)$ and $t \mapsto \varphi(tx, ty)$ imply the continuity of $t \mapsto M(tx)$.

## 1. INTRODUCTION

The concept of stability of a functional equation arises when one replaces a functional equation by an inequality which acts as a perturbation of the equation. In 1940, Ulam [18] posed the first stability problem. In 1941, D.H. Hyers [10] gave the first significant partial solution to his question. Hyers' theorem was generalized for additive mappings by T. Aoki [3] in 1950 and D. G. Bourgin [5] in 1951. In 1978, Th.M. Rassias [17] solved the problem for linear mappings by considering an unbounded Cauchy difference. The phenomenon that was introduced and proved by Th.M.Rassias in the year 1978, is called the Hyers-Ulam-Rassias stability.

Let $X$ and $Y$ be linear spaces and $Y^X$ be the vector space of all functions from $X$ to $Y$. Following [11], for each $x \in X$, define $\Delta_x : Y^X \to Y^X$ by

$$\Delta_x f(y) = f(x+y) - f(y) \quad (f \in Y^X, y \in X).$$

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