

STABILITY OF THE MONOMIAL FUNCTIONAL EQUATION IN
QUASI NORMED SPACES

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ABSTRACT. Let X be a linear space and Y be a complete quasi p -norm space. We will show that for each function $f : X \rightarrow Y$, which satisfies the inequality

$$\|\Delta_n^x f(y)\| \leq \varphi(x, y),$$

for suitable control function φ , there is a unique monomial function M of degree n which is a good approximation for f in such a way that the continuity of $t \mapsto f(tx)$ and $t \mapsto \varphi(tx, ty)$ imply the continuity of $t \mapsto M(tx)$.

1. INTRODUCTION

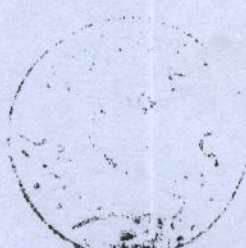
The concept of stability of a functional equation arises when one replaces a functional equation by an inequality which acts as a perturbation of the equation. In 1940, Ulam [18] posed the first stability problem. In 1941, D.H. Hyers [10] gave the first significant partial solution to his question. Hyers' theorem was generalized for additive mappings by T. Aoki [3] in 1950 and D. G. Bourgin [5] in 1951. In 1978, Th.M. Rassias [17] solved the problem for linear mappings by considering an unbounded Cauchy difference. The phenomenon that was introduced and proved by Th.M. Rassias in the year 1978, is called the Hyers-Ulam-Rassias stability.

Let X and Y be linear spaces and Y^X be the vector space of all functions from X to Y . Following [11], for each $x \in X$, define $\Delta_x : Y^X \rightarrow Y^X$ by

$$\Delta_x f(y) = f(x+y) - f(y) \quad (f \in Y^X, y \in X).$$

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