NON-ARCHIMEDEAN STABILITY OF THE MONOMIAL FUNCTIONAL EQUATIONS

ALIREZA KAMEL MIRMOSTAFAEE

ABSTRACT. We study the stability of the functional equation

$$\Delta_x^n f(y) = n! f(x)$$

in non-Archimedean spaces in the spirit of Hyers-Ulam-Rassias-Găvruţa. We will give an example to show that some results in the stability of monomial equations in real normed spaces are not valid in non-Archimedean normed spaces.

1. INTRODUCTION

The following stability is due to S. M. Ulam [25]:
"When is it true that by slightly changing the hypotheses of a theorem one can still assert that the thesis of the theorem remains true or approximately true?" or in other words: Assume that a mathematical object satisfies a certain property approximately according to some convention. Is it then possible to find an object near this object satisfying the property accurately?

In 1941, D.H. Hyers [12] gave the first significant partial solution to this question for additive mappings. Hyers' theorem was generalized by T. Aoki [1] in 1950 for additive mappings. In 1978, Th.M. Rassias [24] solved the problem for linear mappings. In 1994, a generalization of Th.M. Rassias' theorem was obtained by Găvruţa [6], who replaced the bound $\varepsilon(\|x\|^p + \|y\|^p)$ by a general control function $\varphi(x, y)$.

Taking into consideration a lot of influence of Ulam, Hyers and Rassias on the development of stability problems of functional equations, the stability phenomenon that was proved by Th.M. Rassias is called the Hyers–Ulam–Rassias stability. According to P. M.

2000 *Mathematics Subject Classification.* Primary 39B52; Secondary 39B82, 47H10.

Key words and phrases. monomial functional equation; fixed point alternative; Hyers–Ulam–Rassias stability, non-Archimedean normed space

This research was supported by a grant from Ferdowsi University of Mashhad, No:MP88091MIM.