A CHARACTERIZATION OF INNER PRODUCT SPACES CONCERNING AN EULER–LAGRANGE IDENTITY

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ABSTRACT. In this paper we present a new criterion on characterization of real inner product spaces concerning the Euler–Lagrange type identity

$$\|r_2x_1 - r_1x_2\|^2 + \|r_1x_1 + r_2x_2\|^2 = (r_1^2 + r_2^2)(\|x_1\|^2 + \|x_2\|^2)\,.$$

1. INTRODUCTION

In 1932, the notion of (complete) normed linear space was introduced by S. Banach [6]. Then P. Jordan and J. von Neumann [12] showed that a normed linear space V is an inner product space if and only if the parallelogram equality

$$\|x - y\|^2 + \|x + y\|^2 = 2\|x\|^2 + 2\|y\|^2$$

holds for all x and y. Later M.M. Day [9] showed that a normed linear space V is an inner product space if we require only that the parallelogram equality holds for x and y on the unit sphere. In other words, M. M. Day showed that the parallelogram equality may be replaced by the condition $R_2 = 4$ ($\|x\| = 1$, $\|y\| = 1$), where $R_2 = \|x - y\|^2 + \|x + y\|^2$. Over the years, interesting characterizations of inner product spaces have been introduced or developed by numerous mathematicians. Among many significant characterizations for a normed space $V, \|.\|)$ to be inner product we mentioned the

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