STRONG UNIFORM CONSISTENCY OF KERNEL DENSITY ESTIMATORS UNDER CENSORED DEPENDENT MODEL

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ABSTRACT. Problems with censored data arise frequently in survival analyses and reliability applications. The estimation of the density function of the lifetimes is often of interest. In this paper. the estimation of density function by the kernel method is considered, when censored data are showing some kind of dependence. We apply the strong Gaussian approximation technique to study the strong uniform consistency for kernel estimators of the density function under a censored dependent model

1. Introduction and Main Result

In medical follow-up or in engineering life testing studies, one may not be able to observe the variable of interest, referred to hereafter as the lifetime. Let $X_1, \ldots, X_n$ be a sequence of lifetimes, having a common unknown continuous marginal distribution function(d.f.) F, with a density function $f = F'$. The random variables are not assumed to be mutually independent (see Assumption (1) for the kind of dependence stipulated). Let the random variable X_i be censored on the right by the random variable Y_i, so that one observes only

$$Z_i = X_i \wedge Y_i \qquad \text{and} \qquad \delta_i = I(X_i \leq Y_i),$$

where $\wedge$ denotes minimum and $I(.)$ is the indicator of the event specified in parentheses. In this random censorship model, we assume that the censoring random variables $Y_1, \ldots, Y_n$ are not mutually independent (see Assumption (2) for the kind of dependence stipulated), having a common unknown continuous d.f. G, and that they are independent of the X_i's. Since censored data traditionally occur in lifetime analysis, we assume that X_i and Y_i are nonnegative. The actually observed Z_i's have a distribution function H satisfying

$$\overline{H}(t) = 1 - H(t) = (1 - F(t))(1 - G(t)).$$

Denote by

$$F_*(t) = P(Z \leq t, \delta = 1),$$

the sub-distribution function for the uncensored observations, and by f_* the corresponding sub-density. Define

$$N_n(t) = \sum_{i=1}^{n} I(Z_i \leq t, \delta = 1) = \sum_{i=1}^{n} I(X_i \leq t \wedge Y_i),$$

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