



A TRICK FOR INVESTIGATION OF APPROXIMATE DERIVATIONS

M. ESHAGHI GORDJI AND M. S. MOSLEHIAN

ABSTRACT. We utilize the notion of module extension to reduce the problem of stability of derivations to that of ring homomorphisms studied by R. Badora in the context of Banach bimodules over Banach algebras.

1. INTRODUCTION AND PRELIMINARIES

A classical question in the theory of functional equations is that “when is it true that a mapping which approximately satisfies a functional equation \mathcal{E} must be somehow close to an exact solution of \mathcal{E} ?”. Such a problem was formulated by S.M. Ulam [20] in 1940 and solved in the next year for the Cauchy functional equation by D.H. Hyers [9]. It gave rise to the *stability theory* for functional equations. Subsequently, various approaches to the problem have been introduced by several authors. There are cases in which each ‘approximate mapping’ is actually a ‘true mapping’. In such cases, we call the equation \mathcal{E} *superstable*. For the history and various aspects of this theory we refer the reader to monographs [6, 11, 14].

D.G. Bourgin is the first mathematician dealing with the stability of ring homomorphisms. The topic of approximate ring homomorphisms was studied by a number of mathematicians, see [5, 2, 4, 10, 12, 17, 18, 19] and references therein.

2000 *Mathematics Subject Classification*. Primary 39B82; Secondary 39B52, 46H25.

Key words and phrases. Hyers-Ulam stability; derivation; ring homomorphism; Banach algebra; Banach module; module extension.