φ-FRAMES AND φ-RIESZ BASES ON LOCALLY COMPACT ABELIAN GROUPS

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Abstract. We introduce φ-frames in $L^2(G)$, as a generalization of a-frames defined in [7], where G is a locally compact abelian group and φ is a topological automorphism on G. We give a characterization of φ-frames with regard to usual frames in $L^2(G)$ and show that φ-frames share several useful properties with frames. We define the associated φ-analysis and φ-preframe operators, with which we obtain criteria for a sequence to be a φ-frame or a φ-Bessel sequence. We also define φ-Riesz bases in $L^2(G)$ and establish equivalent conditions for a sequence in $L^2(G)$ to be a φ-Riesz basis.

1. Introduction and Preliminaries

The theory of frames was introduced by Duffin and Schaeffer [10] in the early 1950s to deal with problems in nonharmonic Fourier series. There has been renewed interest in the subject related to its role in wavelet theory and a lot of new applications. Several kinds of frames have been introduced up to now; e.g. frames in Hilbert C*-modules (modular frames) [14], frames of subspaces [8], G-frames [26], p-frames [1], frames for Banach spaces [6], a-frames [7], and many others for different purposes. In this paper we define and investigate φ-frames in $L^2(G)$, using the φ-bracket product, as a vector valued inner product on $L^2(G)$ introduced in [19], where G is a locally compact abelian (which will be abbreviated to "LCA ") group and φ is a topological automorphism on G. One of the nice things about φ-frames is the fact that they are useful in studying Gabor systems in the way that there is a close relationship between these frames and Gabor frames in $L^2(G)$. Indeed, our results relate Gabor frames in $L^2(G)$, which have become a paradigm for the spectral analysis associated with time frequency methods [6], to φ-frames. Our construction is related to an extension of Casazza and Lammers' definition of a-frames, $a > 0$, on $L^2(\mathbb{R})$ in [7], to the more general setting of $L^2(G)$, in a new and different approach. We characterize φ-frames in terms of the usual frames in $L^2(G)$ (Theorem 2.1 below), which reveals the above mentioned relation, and we show that φ-frames have several useful properties in common with frames. We also

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