# ON A PROPERTY OF GROUPS WITH COVERINGS

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ABSTRACT. A subgroup $H$ of a group $G$ is conjugately dense in $G$ if for each element $g$ in $G$ the intersection of $H$ with the conjugates of $g$ in $G$ is nonempty. Conjugately dense subgroups deal with interesting open problems, related to parabolic groups. In the present paper we study them with respect to suitable coverings.

## 1. INTRODUCTION

If $H$ is a subgroup of a group $G$, $H$ is said to be *conjugately dense* in $G$ if for each element $g$ of $G$ the intersection of $H$ with the conjugates of $g$ in $G$ is nonempty. These subgroups arouse interest in various situations. For instance, [34, Problem 8.8b] is an open question on the existence of a noncyclic finitely presented group with a conjugately dense cyclic subgroup. It is conjectured that each conjugately dense irreducible subgroup of $GL(n, K)$, the general linear group of degree $n$ over an arbitrary field $K$, where $n$ is a fixed positive integer, coincides always with the whole group $GL(n, K)$, except when $K$ is quadratically closed, $n = 2$ and the characteristic of $K$ is 2. There is not too much literature on the topic: mainly [23, 24, 39, 40, 41].

The validity of [34, Problem 8.8b] in the general case would imply that each conjugately dense subgroups of $GL(n, K)$ is parabolic, but [41, Theorem 2] shows that this is not true. From the argument of [41, Theorem 2], a positive answer to [34, Problem 8.8b] depends on a hand by the choice of $K$, by the characteristic of $K$ and by $n$, on an other hand by the properties of stability of the conjugately dense subgroups of $GL(n, K)$ with respect to certain usual operations between subgroups such as intersections, products and conjugation. A big problem in working with conjugately dense subgroups is due to the fact that they do not form a class of groups closed with respect to subgroups and subdirect products. This difficulty has been often mentioned in [23, 24, 39, 40], noting that the intersection of two conjugately dense subgroups of a group $G$ can not be a conjugately dense subgroup of $G$. Here we will study conjugately dense subgroups in some specific contexts.

The main results of the present paper have been proved in successive steps. Section 2 recalls the definitions of classes of generalized $FC$-groups. Section 3 gives some notions in theory of groups with coverings. Then we can formulae the main results of the present paper in Section 4. Section 5 is devoted to general properties of the conjugately dense subgroups. Finally, the proofs of the main results of this paper are contained in Section 6.

Most of our notation is standard and referred to [8] and [30]. The properties of conjugately dense subgroups are referred to [6, 23, 24, 39, 40, 41]. The properties of generalized $FC$-groups are referred to [9, 10, 19, 20, 28, 29, 30]. The properties of groups with coverings are referred to [12, 13, 15, 16, 31, 32, 33].

## 2. SOME CLASSES OF GENERALIZED $FC$-GROUPS

Let $\mathfrak{X}$ be a class of groups. An element $x$ of a group $G$ is said to be an $\mathfrak{X}C$-*element* if $G/C_G(\langle x\rangle^G)$ satisfies $\mathfrak{X}$. A group whose elements are all $\mathfrak{X}C$-elements is said to be an

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