(σ, τ)-AMENABILITY OF C^*-ALGEBRAS

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ABSTRACT. Suppose that $\mathcal{A}$ is an algebra, $\sigma, \tau : \mathcal{A} \to \mathcal{A}$ are two linear mappings such that both $\sigma(\mathcal{A})$ and $\tau(\mathcal{A})$ are subalgebras of $\mathcal{A}$ and $\mathcal{X}$ is a $(\tau(\mathcal{A}), \sigma(\mathcal{A}))$-bimodule. A linear mapping $D : \mathcal{A} \to \mathcal{X}$ is called a (σ, τ)-derivation if $D(ab) = D(a) \cdot \sigma(b) + \tau(a) \cdot D(b)$ $(a, b \in \mathcal{A})$. A (σ, τ)-derivation D is called a (σ, τ)-inner derivation if there exists an $x \in \mathcal{X}$ such that D is of the form either $D_x^-(a) = x \cdot \sigma(a) - \tau(a) \cdot x$ $(a \in \mathcal{A})$ or $D_x^+(a) = x \cdot \sigma(a) + \tau(a) \cdot x$ $(a \in \mathcal{A})$. A Banach algebra $\mathcal{A}$ is called (σ, τ)-amenable if every (σ, τ)-derivation from $\mathcal{A}$ into a dual Banach $(\tau(\mathcal{A}), \sigma(\mathcal{A}))$-bimodule is (σ, τ)-inner. Studying some general algebraic aspects of (σ, τ)-derivations, we investigate the relation between amenability and (σ, τ)-amenability of Banach algebras in the case when σ, τ are homomorphisms. We prove that if $\mathfrak{A}$ is a C^*-algebra and σ, τ are $*$-homomorphisms with $\ker(\sigma) = \ker(\tau)$, then $\mathfrak{A}$ is (σ, τ)-amenable if and only if $\sigma(\mathfrak{A})$ is amenable.

1. INTRODUCTION AND PRELIMINARIES

The notion of an amenable Banach algebra was introduced by B.E. Johnson in his definitive monograph [7]. This class of Banach algebras arises naturally out of the cohomology theory for Banach algebras, the algebraic version of which was developed by G. Hochschild [6]. For a comprehensive account on amenability the reader is referred to books [4, 13].

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$$D(ab) = D(a) \cdot \sigma(b) + \tau(a) \cdot D(b) \qquad (a, b \in \mathcal{A}).$$

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