



## Optimal sample size for record data and associated cost analysis for exponential distribution

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(Received 12 April 2009; final version received 29 June 2009)

Estimation of the mean of an exponential distribution based on record data has been treated by Samaniego and Whitaker [F.J. Samaniego, and L.R. Whitaker, *On estimating popular characteristics from record breaking observations I. Parametric results*, Naval Res. Logist. Quart. 33 (1986), pp. 531–543] and Doostparast [M. Doostparast, *A note on estimation based on record data*, *Metrika* 69 (2009), pp. 69–80]. When a random sample  $Y_1, \dots, Y_n$  is examined sequentially and successive minimum values are recorded, Samaniego and Whitaker [F.J. Samaniego, and L.R. Whitaker, *On estimating popular characteristics from record breaking observations I. Parametric results*, Naval Res. Logist. Quart. 33 (1986), pp. 531–543] obtained a maximum likelihood estimator of the mean of the population and showed its convergence in probability. We establish here its convergence in mean square error, which is stronger than the convergence in probability. Next, we discuss the optimal sample size for estimating the mean based on a criterion involving a cost function as well as the Fisher information based on records arising from a random sample. Finally, a comparison between complete data and record is carried out and some special cases are discussed in detail.

**Keywords:** exponential model; life-testing experiment; record values; maximum likelihood estimator; Fisher information; optimal sample size; cost function

### 1. Introduction

In reliability, we are concerned primarily with test data in which lifetimes of items that fail during the course of the test are recorded, or with variables related in some way to item lifetimes. If the actual lifetime of every item in the sample is recorded, the data are *complete* data. To obtain complete data, it is necessary to continue the experiment until the last item on test or in service has failed. In cases where even a few items in the sample may have very long lifetimes, the experiment can go on for a very long period of time and, in fact, well beyond the point at which the results may no longer be of any interest or use. In such situations, it may be desirable to terminate the study prior to failure of all items under test. When observation is discontinued prior to all items

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