AN APPLICATION OF BANACH'S FIXED POINT THEOREM TO THE STABILITY OF A GENERAL FUNCTIONAL EQUATION

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ABSTRACT. Using the Banach fixed point theorem we establish the stability of the functional equation $f(z) = G\big(f(\psi(z)), f(\varphi(z))\big)$, where f is an unknown function, under certain natural assumptions on functions G, φ and ψ. Our main result seems to be an extension of a known one of J. Baker.

1. INTRODUCTION

We say a functional equation $(\mathcal{E})$ is stable if any function which approximately satisfies $(\mathcal{E})$ is near to an exact solution of the equation $(\mathcal{E})$. More than a half century ago, S.M. Ulam [13] proposed the first stability problem which was partially solved by D.H. Hyers [5] in the framework of Banach spaces. Since then, many mathematicians worked on this live area of research, see monographs [4, 6, 8, 11]. There are four methods in the study of stability of functional equations. The first method is the *direct method* in which one uses an iteration process producing the so-called *Hyers type sequences* [5]. Another method is based on *sandwich theorems* which are generalizations of the Hahn-Banach separation theorem; cf. [10]. The third technique focuses on using *invariant means*; cf. [12], and the foundation of the forth method is *fixed point techniques*; cf. [3] (see also [7, 9]). In this paper, using some ideas from [2] and [3], we establish the stability of the functional equation

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