HYERS-ULAM STABILITY OF CUBIC MAPPINGS IN NON-ARCHIMEDEAN NORMED SPACES

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ABSTRACT. We give a fixed point approach to the generalized Hyers-Ulam stability of the cubic equation

$$f(2x+y)+f(2x-y)=2f(x+y)+2f(x-y)+12f(x)$$

in non-Archimedean normed spaces. We will give an example to show that some known results in the stability of cubic functional equations in real normed spaces fail in non-Archimedean normed spaces. Finally, some applications of our results in non-Archimedean normed spaces over p-adic numbers will be exhibited.

1. INTRODUCTION

The concept of stability of a functional equation arises when one replaces a functional equation by an inequality which acts as a perturbation of the equation. In 1940, S.M. Ulam [28] posed the first stability problem. In the next year, D. H. Hyers [8] gave a partial affirmative answer to the Ulam's problem. The theorem of Hyers was generalized by T. Aoki [1] and Bourgin [3]. In 1978, Th. M. Rassias [27] provided a remarkable generalization of Hyers's result by allowing the Cauchy difference to be unbounded. In 1994, a generalization of Rassias' theorem was obtained by P. Găvruta [6] by replacing the bound $\varepsilon(\|x\|^p+\|y\|^p)$ by a general control function $\varphi(x,y)$. Several stability results have been recently obtained for various equations, also for mappings with more general domains and ranges (see e.g. [9, 10]).

The functional equation

$$f(2x+y)+f(2x-y)=2f(x+y)+2f(x-y)+12f(x) \tag{1.1}$$

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