DUNKL–WILLIAMS INEQUALITY FOR OPERATORS ASSOCIATED WITH p-ANGULAR DISTANCE

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ABSTRACT. We present several operator versions of the Dunkl–Williams inequality with respect to the p-angular distance for operators. More precisely, we show that if $A, B \in \mathbb{B}(\mathscr{H})$ such that $|A|$ and $|B|$ are invertible, $\frac{1}{r}+\frac{1}{s}=1$ $(r>1)$ and $p \in \mathbb{R}$, then

$$|A|A|^{p-1}-B|B|^{p-1}|^2 \leq |A|^{p-1}(r|A-B|^2+s||A|^{1-p}|B|^p-|B||^2)|A|^{p-1}.$$

In the case that $0<p\leq 1$, we remove the invertibility assumption and show that if $A=U|A|$ and $B=V|B|$ are the polar decompositions of A and B, respectively, $t>0$, then

$$|(U|A|^p-V|B|^p)|A|^{1-p}|^2 \leq (1+t)|A-B|^2+(1+\frac{1}{t})||B|^p|A|^{1-p}-|B||^2.$$

We obtain several equivalent conditions, when the case of equalities hold.

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