Piecewise-Truncated Parametric Iteration Method: a Promising Analytical Method for Solving Abel Differential Equations

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This paper deals with the analytical approximate solution of Abel differential equations (ADEs) of the type $du/dt = \sum_{k=0}^{m} g_k(t)u^k$ by proposing a new modified version of the parametric iteration method (PIM). The modified algorithm analytically approximates the solution of ADEs in a sequence of subintervals which is continuous everywhere. The local convergence and the stability of the algorithm are discussed in details. Also we show how to characterize the stability function and the region on which the algorithm is presented. Some examples are given to demonstrate the efficiency and accuracy of this algorithm. Comparison with numerical Runge-Kutta methods (RK) shows that the modified algorithm presented in this paper has the advantage of giving an analytical form of the solution, which is not possible in the purely numerical RK techniques. Moreover, the approximations obtained by the new algorithm converge faster than the numerical RK4 methods, as will be shown. The obtained results reveal that the present algorithm is a promising iterative analytical method for solving ADEs. Furthermore, the proposed algorithm provides us with an easy way to modify the convergence region and rate of the solution. Most promising, however, it seems that the newly developed technique can be further implemented easily to solve other nonlinear ordinary differential equations (ODEs) of physical interests.

Key words: Piecewise-Truncated Parametric Iteration Method; Truncated Parametric Iteration Method; Parametric Iteration Method; Abel Differential Equations; Runge-Kutta Methods
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1. Introduction

It is well known that many phenomena in scientific fields can be described by nonlinear ordinary differential equations. These nonlinear models of real-life problems are still difficult to reliably solve either numerically or theoretically. And even if an exact solution is obtainable, the required calculations may be too complicated to be practical, or the resulting solution may be difficult to interpret. In recent times, there has been much attention devoted to search for better and more efficient solution methods for determining a solution, approximate or exact, analytical or numerical, to these nonlinear models (please see [1 – 10] and the references cited therein). In this direction, recently, the new analytical approximate methods such as the Adomian decomposition method (ADM) [3], the homotopy analysis method (HAM) [5], the variational iteration method (VIM) [7], the homotopy perturbation method (HPM) [9], the parametric iteration method (or fractional iteration method) (PIM) [1] have been proposed to effectively handle nonlinear problems. In this work, we consider the Abel differential equation of the first kind of the form

$$u'(t) = \sum_{k=0}^{m} g_k(t)u^k(t), \quad u(t_0) = c, \tag{1}$$

where $g_k(t)$ $(k = 0, 1, \dots, m)$ are given analytic functions, c is a constant, and u is the solution to be determined later. These differential equations arose in the context of the studies of N. H. Abel [11] on the theory of elliptic functions. An interesting area of this type of equation also arises in biological systems, see [12], and also in physics, engineering, ecology, and economics (see [13]). Thus, methods to solve these equations are of great importance to engineers and scientists. It represents a natural generalization of the well-known Riccati equation. The solution of this equation can be obtained using numerical integration methods. Although

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