SOME CONCEPTS OF NEGATIVE DEPENDENCE FOR BIVARIATE DISTRIBUTIONS WITH APPLICATIONS

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ABSTRACT. In this paper, some concepts of negative dependence for bivariate distributions, especially hazard and local negative dependence (HND, LND) are studied. The Clayton-Oakes, φ and γ measures of association and relationship of HND with these measures are obtained. In addition, various examples illustrate the usefulness of these notions in some family of distributions.

AMS Subject Classification: Primary 62H20; Secondary 62E99.

Keywords and Phrases: Local dependence function, Hazard negative dependence, Local negative dependence, Revers rule of order2 (RR_2), $RCSD$, Copula function.

1. INTRODUCTION AND PRELIMINARIES

Let X and Y be absolutely continuous random variables having joint density $f(x,y)$ and survival function $\bar{F}(x,y)$. Basu [3] introduced bivariate hazard function by $r(x,y) = f(x,y)/\bar{F}(x,y)$. In the independent case the bivariate hazard function is equal to product of conditional hazard functions, $\frac{\partial}{\partial x}[-\log \bar{F}(x,y)]$ and $\frac{\partial}{\partial y}[-\log \bar{F}(x,y)]$. If equality failed we deal with dependent (positive or negative) random variables. Oluyede [17] and [18] has obtained some properties and inequalities for positively hazard and local dependence. More details about notions of dependence are in Lehmann [14], Karlin [13], Esary and Proschan [5], Joe [10] and Shaked and Shanthikumar [20]. In this paper we use notions of negatively hazard and local dependence, say HND, LND, and investigate relationship between these concepts with some other concepts of dependence. Finally, we obtain measures of association as Θ-measure (known as Clayton-Oakes measure), φ-measure and γ-measure for some bivariate distributions family, then evaluate the relationship between these measures and $HND(LND)$.

Let (X,Y) be an absolutely continuous random vector with distribution function $F(x,y)$ and survival function$\bar{F}(x,y)$. Next, we need the following definitions.

DEFINITION 1.1. ([17]) Absolutely continuous random variables X and Y having a joint density function $f(x,y)$ are hazard negative (positive)dependence, HND(HPD). if

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THIS RESEARCH WAS SUPPORTED BY A GRANT FROM FERDOWSI UNIVERSITY OF MASHHAD (NO. MS89140AMI).