The pair of finite p-groups

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Abstract

Let M be a normal subgroup of a group G such that $|G/M| = p^m$ and $|M/Z(M,G)| = p^n$. In this paper, assuming $|[M,G]| = p^{\frac{1}{2}n(n+2m-1)-1}$ and $|[M/Z(M,G), G/Z(M,G)]| \leq p$, we characterize all the pairs of groups (M,G) which satisfy these conditions.

If M has a complement in G and $|M| = p^n$ and $|G/M| = p^m$, then there always exists a non-negative integer $t(M,G)$ such that $|\mathcal{M}(M,G)| = p^{\frac{1}{2}n(n+2m-1)-t(M,G)}$. Now, under some conditions, we determine all the pairs (M,G) with $t(M,G) = 0, 1$ or 2.

1 Introduction

Let (M,G) be a pair of groups, in which M is (isomorphic to) a normal subgroup of G. Let $[M,G]$ and $Z(M,G)$ denote the commutator subgroup and the centralizer of G in M,

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