JEWELL THEOREM FOR HIGHER DERIVATIONS ON C^*-ALGEBRAS

SHIRIN HEJAZIAN, MADJID MIRZAVAZIRI AND ELAHE OMIDVAR TEHRANI

ABSTRACT. Let $\mathcal{A}$ be an algebra. A sequence $\{d_n\}$ of linear mappings on $\mathcal{A}$ is called a higher derivation if $d_n(ab) = \sum_{j=0}^{n} d_j(a)d_{n-j}(b)$ for each $a, b \in \mathcal{A}$ and each nonnegative integer n. Jewell [Pacific J. Math. **68** (1977), 91-98], showed that a higher derivation from a Banach algebra onto a semisimple Banach algebra is continuous provided that $\ker(d_0) \subseteq \ker(d_m)$, for all $m \geqslant 1$. In this paper, under a different approach using C^*-algebraic tools, we prove that each higher derivation $\{d_n\}$ on a C^*-algebra $\mathfrak{A}$ is automatically continuous, provided that it is normal, i.e. d_0 is the identity mapping on $\mathfrak{A}$.

1. INTRODUCTION

Let $\mathcal{A}$ be an algebra. A linear mapping $\delta : \mathcal{A} \to \mathcal{A}$ is called a derivation if it satisfies the Leibniz rule, i.e. $\delta(ab) = \delta(a)b + a\delta(b)$ for all $a, b \in \mathcal{A}$. If we define the sequence $\{d_n\}$ of linear mappings on $\mathcal{A}$ by $d_0 = I$ and $d_n = \frac{\delta^n}{n!}$, where I is the identity mapping on $\mathcal{A}$, then the Leibniz rule ensures us that d_n's satisfy the condition

$$d_n(ab) = \sum_{j=0}^{n} d_j(a)d_{n-j}(b) \quad (*)$$

for each $a, b \in \mathcal{A}$ and each nonnegative integer n. This motivates us to consider the sequences $\{d_n\}$ of linear mappings on an algebra $\mathcal{A}$ satisfying $(*)$. Such a sequence is called a higher derivation. Higher derivations were introduced by Hasse and Schmidt [2], and algebraists sometimes call them Hasse-Schmidt derivations. Though, if $\delta : \mathcal{A} \to \mathcal{A}$ is a derivation then $d_n = \frac{\delta^n}{n!}$ is a higher derivation, this is not the only example of a higher derivation.

Regarding to a celebrated theorem of Sakai [11, 12], all derivations defined on a C^*-algebra are automatically continuous. Some results concerning to the theorem are discussed in [8] and [3]. Regarding to the Sakai's theorem we can deduce that the higher derivation $d_n = \frac{\delta^n}{n!}$ defined on a C^*-algebra is automatically continuous in the sense that

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