SHIFT PRESERVING OPERATORS ON LOCALLY COMPACT ABELIAN GROUPS

R.A. KAMYABI GOL AND R. RAISI TOUSI

ABSTRACT. We investigate shift preserving operators on locally compact abelian groups. We show that there is a one-to-one correspondence between shift preserving operators and range operators on $L^2(G)$ where G is a locally compact abelian group. We conclude that a shift preserving operator has several properties in common with its associated range operator, especially compactness of one implies compactness of the other. Moreover, we obtain a necessary condition for a shift preserving operator to be Hilbert Schmidt or of finite trace in terms of its range function.

1. INTRODUCTION AND PRELIMINARIES

A bounded linear operator $U : L^2(\mathbb{R}^n) \to L^2(\mathbb{R}^n)$ is called shift preserving (which will be abbreviated to "SP") if $UT_k = T_kU$ for all $k \in \mathbb{Z}^n$, where T_k is the shift operator. As a special case of a shift operator is the time delay operator $T_k : l^2 \to l^2$ defined by $T_k u(n) = u(n-k)$, $u \in l^2$, $k.\ n \in \mathbb{Z}$ where the action is to delay the signal u by k units. A digital filter U is a SP operator on l^2. In other words a filter is a time invariant operator in which delaying the input by k units of time is just to delay the output by k units. These operators play an important role in signal processing, such as to analyse, code, reconstruct signals and so on. They are often used to extract required frequency components from signals. For example, high frequency components of a signal usually contain the noise and fluctuations, which often have to be removed from the signal using different kinds of filters. For more details and examples of filters cf. [10, 4].

SP operators on $\mathbb{R}^n$ have been studied by Bownik in [3]. He gave a characterization of these operators in terms of range operators. Our goal in this paper is to investigate SP operators on locally compact abelian (which will be abbreviated to "LCA") groups. The major result in this paper is a novel characterization of SP operators on $L^2(G)$, where G is a LCA group. This allows us to handel SP operators (specially filters) on

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