Almost sure convergence of kernel bivariate distribution function estimator under negative association*

H. Jabbari †

Department of Statistics, Ordered and Spatial Data Center of Excellence,

Ferdowsi University of Mashhad, Mashhad, Iran.

Abstract

Let $\{X_n, n \geq 1\}$ be a strictly stationary sequence of negatively associated random variables, with common distribution function F. In this paper, we consider the estimation of the two-dimensional distribution function of (X_1, X_{k+1}) for fixed $k \in I\!N$ based on kernel type estimators. We introduce asymptotic normality and properties and moments. From these we derive the optimal bandwidth convergence rate, which is of order n^{-1}. Besides of some usual conditions on the kernel function, the conditions typically impose a convenient increase rate on the covariances $Cov(X_1, X_n)$.

MSC 2000: Primary 62G20; Secondary 60F15, 62N05.

keywords: Almost sure convergence, Bivariate distribution function, Kernel estimation, Negatively association, Stationarity.

1 Introduction, definitions and assumptions

The interest on approximating distribution functions of random pairs arises from the characterizations of the limiting distribution of empirical processes, which has been a subject of interest for many statisticians. The first results concerning the asymptotic distribution of the sequence data back to Donsker [3], for independent underlying variables $\{X_n, n \geq 1\}$. The extension of this characterization to non-independent variables was eventually studied. One of the dependence structures is positive association. Azevedo and Oliveira [2] studied kernel type estimation of bivariate distribution function for positively associated random variables. The other type of dependence is negative association (NA), introduced by Alam and Saxena [1] and carefully studied by Joag-Dev and Proschan [6]. A finite family of random variables $\{X_i, 1 \leq i \leq n\}$ is said to be negatively associated if for every pair of disjoint subsets A and B of $\{1, 2, ..., n\}$,

$$Cov(f_1(X_i, i \in A), f_2(X_j, j \in B)) \leq 0$$

whenever f_1 and f_2 are coordinatewise increasing and such that the covariance exists. An infinite family of random variables is NA if every finite subfamily is NA. Because of their wide applications in multivariate statistical analysis and reliability theory, the notion of NA has received more and more attention recently. We refer to Joag-Dev and Proschan [6] for fundamental properties, Newman [9] and Su and Chi [14] for central limit theorem, Matula [8] for three series theorem, Su et al. [15] for a moment inequality, a weak invariance principle and example to show that there exists infinite family of non-degenerate non-independent strictly stationary NA random variables, Shao [13] for the Rosenthal type maximal inequality and Kolmogorov exponenential inequality, Liang and Su [7] for convergence rates of law of the logarithm, Roussas [11] for the central limit theorem of random fields, some examples and applications and Yuan et al. [16] for improving the result of Roussas [11]. The

*The research was supported by a grant from Ferdowsi University of Mashhad (No. MS88124JAB).
†email: Jabbarinh@um.ac.ir

146