



Optimal record-based statistical procedures for the two-parameter exponential distribution

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Abstract

There are a number of situations in which the experimental data observed are record statistics. In this paper, optimal confidence intervals as well as uniformly most powerful tests for one-sided alternatives are developed. Since there does not exist a uniformly most powerful test for a two-sided alternative, generalized likelihood ratio and uniformly unbiased and invariant tests are derived for the two parameters of the exponential distribution based on record data. For illustrative purposes, a data set on the times between consecutive telephone calls to a company's switchboard is analyzed using the proposed procedures. Finally, some open problems in this direction are pointed out.

Keywords and phrases: Two-parameter exponential model; Generalized likelihood ratio test; Invariant test; Monotone likelihood ratio; Neyman-Pearson fundamental lemma; Shortest-width confidence interval; Uniformly most powerful test; Unbiased interval.

1 Motivation

Let X_1, X_2, X_3, \dots be a sequence of continuous random variables. X_k is a lower record value if it is smaller than all preceding values X_1, X_2, \dots, X_{k-1} . By definition, X_1 is taken as the first lower record value. An analogous definition can be provided for upper record values. Record statistics arise naturally in many practical problems and in applied fields such as weather and sports wherein a particular data point is archived for posterity if it is smaller or larger than all others recorded in the past, but perhaps is not stored so accessibly otherwise. Indeed, data that are not records are sometimes not available at all.

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