

ASYMPTOTIC ASPECT OF THE QUADRATIC FUNCTIONAL EQUATION IN MULTI-NORMED SPACES

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ABSTRACT. We investigate the Hyers–Ulam stability of the quadratic functional equation for mappings from abelian groups into multi-normed spaces. We also study the stability on a restricted domain and present an asymptotic behavior of the quadratic equation in the framework of multi-normed spaces.

1. INTRODUCTION AND PRELIMINARIES

In 1940, S.M. Ulam [27] posed the first stability problem for linear mappings. In the next year, D.H. Hyers [12] gave an affirmative answer to the question of Ulam. In 1950, T. Aoki [1] proved a more general result, which was next rediscovered by Th.M. Rassias in 1978, [24] and from then it is called the Hyers–Ulam–Rassias stability theorem (see [7]). During the last decades several stability problems for various functional equations have been investigated by numerous mathematicians (cf. e.g. [6, 8, 11, 13, 17, 21] and the bibliography quoted there).

The functional equation

$$f(x + y) + f(x - y) = 2f(x) + 2f(y)$$

is called the *quadratic functional equation*. In particular, every solution Q of the quadratic functional equation is said to be a *quadratic mapping*. The Hyers–Ulam stability problem for the quadratic functional equation was solved by F. Skof [26] (for mappings $f : \mathcal{X} \rightarrow \mathcal{Y}$, where \mathcal{X} is a normed

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