

A CLASS OF DISTAL FUNCTIONS ON SEMITOPOLOGICAL SEMIGROUPS

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ABSTRACT. The norm closure of the algebra generated by the set $\{n \mapsto \lambda^{n^k} : \lambda \in \mathbb{T} \text{ and } k \in \mathbb{N}\}$ of functions on $(\mathbb{Z}, +)$ was studied in [9] (and was named as the Weyl algebra). In this paper, by a fruitful result of Namioka, this algebra is generalized for a general semitopological semigroup and, among other things, it is shown that the elements of the involved algebra are distal. In particular, we examine this algebra for $(\mathbb{Z}, +)$ and (more generally) for the discrete (additive) group of any countable ring. Finally, our results are treated for a bicyclic semigroup.

1. INTRODUCTION

Distal functions on topological groups were extensively studied by Knapp [5]. The norm closure of the algebra generated by the set $F = \{n \mapsto \lambda^{n^k} : \lambda \in \mathbb{T} \text{ and } k \in \mathbb{N}\}$ of functions on $(\mathbb{Z}, +)$ was called the Weyl algebra by E. Salehi in [9]. Knapp, [5], showed that all of the elements of F are distal on $(\mathbb{Z}, +)$. Also Namioka [7, Theorem 3.6] proved the same result by using a very fruitful result ([7, Theorem 3.5]) which played an important role for the construction of this paper. By the above mentioned results of Knapp and Namioka, all elements of the Weyl algebra are distal, however it does not exhaust all distal functions on $(\mathbb{Z}, +)$; [9, Theorem 2.14]. In this paper, we generalize the notion of Weyl algebra to an arbitrary semitopological semigroup and also we show that all elements of the involved algebra are distal. In particular, our method provides a convenient way to deduce a result of M. Filali [3] on the distality of the functions $\chi(q(t))$, where χ is a character on the discrete additive group of a (countable) ring R and $q(t)$ is a polynomial with coefficients in R .

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