INNERNESS OF ρ-DERIVATIONS ON HYPERFINITE VON NEUMANN ALGEBRAS

MADJID MIRZAVAZIRI AND MOHAMMAD SAL MOSLEHIAN

ABSTRACT. Suppose that $\mathfrak{M}, \mathfrak{N}$ are von Neumann algebras acting on a Hilbert space and $\mathfrak{M}$ is hyperfinite. Let $\rho : \mathfrak{M} \to \mathfrak{N}$ be an ultraweakly continuous $*$-homomorphism and let $\delta : \mathfrak{M} \to \mathfrak{N}$ be a $*$-ρ-derivation such that $\delta(I)$ commutes with $\rho(I)$. We prove that there is an element U in $\mathfrak{N}$ with $\|U\| \le \|\delta\|$ such that $\delta(A) = U\rho(A) - \rho(A)U$ for all $A \in \mathfrak{M}$.

1. Introduction

The theory of algebras of operators on Hilbert spaces started around 1930 with papers of von Neumann and Murray. The principal motivations of these authors were the theory of unitary group representations and certain aspects of the quantum mechanical formalism. The von Neumann algebras are significant for mathematical physics since the most fruitful algebraic reformulation of quantum statistical mechanics and quantum field theory was in terms of these algebras, cf. [1, 2]. The study of theory of derivations in operator algebras is motivated by questions in quantum physics and statical mechanics. One of important questions in the theory of derivations is that " When are all bounded derivations inner?" Forty years ago, R.V. Kadison [8] and S. Sakai [15] independently proved that every derivation of a von Neumann algebra $\mathfrak{M}$ into itself is inner, see also papers of [7, 9]. This was the starting point for the study of the so-called bounded cohomology groups (see [13]). This nice result can be restated as saying that the first cohomology group $H^1(\mathfrak{M}; \mathfrak{M})$ (i.e. the vector space of derivations modulo the inner derivations) vanishes.

Let $\mathfrak{A}$ and $\mathfrak{B}$ be two algebras, $\mathfrak{X}$ be a $\mathfrak{B}$-bimodule and $\rho : \mathfrak{A} \to \mathfrak{B}$ be a homomorphism. A linear mapping $\delta : \mathfrak{A} \to \mathfrak{X}$ is called a ρ-derivation if $\delta(ab) = \delta(a)\rho(b) + \rho(a)\delta(b)$ for all $a, b \in \mathfrak{A}$. These maps have been extensively investigated in pure algebra. Recently,

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