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On the Order of Polynilpotent Multipliers of Some Nilpotent Products of Cyclic p-Groups

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Abstract. In this article we show that if $\mathcal{V}$ is the variety of polynilpotent groups of class row $(c_1, c_2, ..., c_s)$, $\mathcal{N}_{c_1,c_2,...,c_s}$, and $G \cong \mathbf{Z}_{p^{\alpha_1}} \overset{n}{*} \mathbf{Z}_{p^{\alpha_2}} \overset{n}{*} ... \overset{n}{*} \mathbf{Z}_{p^{\alpha_t}}$ is the nth nilpotent product of some cyclic p-groups, where $c_1 \geq n$, $\alpha_1 \geq \alpha_2 \geq ... \geq \alpha_t$ and $(q,p) = 1$ for all primes q less than or equal to n, then $|\mathcal{N}_{c_1,c_2,...,c_s}M(G)| = p^{d_m}$ if and only if $G \cong \mathbf{Z}_p \overset{n}{*} \mathbf{Z}_p \overset{n}{*} ... \overset{n}{*} \mathbf{Z}_p$ (m-copies), where $m = \sum_{i=1}^{t} \alpha_i$ and $d_m = \chi_{c_s+1}(...(\chi_{c_2+1}(\sum_{j=1}^{n} \chi_{c_1+j}(m)))...)$. Also, we extend the result to the multiple nilpotent product $G \cong \mathbf{Z}_{p^{\alpha_1}} \overset{n_1}{*} \mathbf{Z}_{p^{\alpha_2}} \overset{n_2}{*} ... \overset{n_{t-1}}{*} \mathbf{Z}_{p^{\alpha_t}}$, where $c_1 \geq n_1 \geq ... \geq n_{t-1}$. Finally a similar result is given for the c-nilpotent multiplier of $G \cong \mathbf{Z}_{p^{\alpha_1}} \overset{n}{*} \mathbf{Z}_{p^{\alpha_2}} \overset{n}{*} ... \overset{n}{*} \mathbf{Z}_{p^{\alpha_t}}$ with the different conditions $n \geq c$ and $(q,p) = 1$ for all primes q less than or equal to $n + c$.

Keywords: Polynilpotent multiplier; Nilpotent product; Cyclic group; Finite p-group; Elementary Abelian p-group.

AMS Subject Classifications: 20C25, 20D15, 20E10, 20F18, 20F12.

1 Introduction and motivation

Let G be any group with a presentation $G \cong F/R$, where F is a free group. Then the Baer invariant of G with respect to the variety of groups $\mathcal{V}$, denoted by $\mathcal{V}M(G)$, is defined to be

$$\mathcal{V}M(G) = \frac{R \cap V(F)}{[RV^*F]},$$

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